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## Estimation of a Speed Hump Profile Using Quarter Car Model

Apichan Kanjanavapastit<sup>a,\*</sup>, Aphirak Thitinaruemit<sup>b</sup>

<sup>a</sup>Department of Electronics Engineering, Faculty of Technology, Udon Thani Rajabhat University, Udon Thani, 41000, Thailand

<sup>b</sup>Faculty of Industrial Technology Suan Sunandha Rajabhat University, Bangkok, 10300, Thailand

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### Abstract

In the past few years, the topics concerning vehicular communications have been in conducting researches. One research topic is to acquire important information (e.g., traffic conditions, weather conditions) necessary to disseminate to car drivers. Road profile information is very important information that the drivers who know this in advance can use to avoid accidents. Speed hump on a road, which is one of the road profile information, can cause an accident especially at night if a driver has never passed through the road before. This paper proposes a technique to estimate speed hump profile using quarter car model. In this technique, two acceleration sensors are needed to install at sprung mass and unsprung mass of a car. The signals of the two sensors are then passed through the estimation model to get the speed hump profile in time unit (i.e., height versus time). To obtain the speed hump profile in space unit (i.e., height versus distance), the information of car velocities while passing the speed hump is required. Simulation results from Simulink confirm that the proposed technique can estimate speed hump profile accurately.

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### 1. Introduction

Vehicle communication technology is gaining much attention from researchers due to its benefits such as reducing traffic accidents. For example, when a car has detected that a section of a road is too slippery and not suitable for fast driving, the car can send out information to tell nearby cars; then the drivers of those nearby cars can be careful when driving through the section. Road profile information is very important information that the drivers who know this in advance can avoid accidents. There were a number of techniques concerned about getting a road profile such as stereo technique [1], laser scanning technique [2], and template matching technique

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\* Corresponding author. E-mail address: [apichank@hotmail.com](mailto:apichank@hotmail.com)

[3]. The stereo technique needs high computational complexity, but the laser scanning technique needs expensive equipment (i.e., laser scanner). The template matching technique, which was our previous work, uses low computational complexity and normal computing equipment. However, it gives crude information of the road profile which is merely the existence of speed humps. More information of the speed hump profile, such as height and width, are therefore needed.

Mathematical car model can be used to obtain the road information. For example, the authors in [4] proposed a classification of different road surfaces (e.g., asphalts and bricks) by installing acceleration sensor at unsprung mass. Then the signals from the sensor were analyzed using quarter car model, and the road surfaces were classified using Support Vector Machine. In addition, the authors in [5] simulated the operation of a car when passing road humps to find the ride quality. The car was modeled using two dimensional half-car models which are more complex than the quarter car model. As seen from the last two references, the method of using a car model is very interesting. This paper, therefore, proposes the use of quarter car model to estimate the speed hump profile. In this technique, two acceleration sensors are needed to install at sprung mass and unsprung mass of a car. The signals of the two sensors are then passed through the proposed estimation model to get the speed hump profile in time unit (i.e., height versus time). To obtain the speed hump profile in space unit (i.e., height versus distance), the information of car velocities while passing the speed hump is required.

## 2. Quarter Car Model

The quarter car model is the model that considers the movement of a car with suspension in vertical direction. The model is widely used by researchers since it is simple and of low complexity. As seen in figure 1, the model consists of two different masses and two sets of spring and damper. Sprung mass,  $m_s$ , is considered only a quarter of the total mass supported above the suspension. Therefore, the sprung mass includes, for example, car body, seats, internal components, and passengers. Unsprung mass,  $m_{us}$ , is also considered a quarter of the total mass suspended below the suspension. The unsprung mass then includes, for example, wheels, wheel bearings, brake rotors, and drive shaft. The road height, that the wheel is contacted with, versus time is  $y_{road}$ . Then,  $\dot{y}_{road}$  is the rate of the road height which is therefore the vertical velocity at the wheel,  $v_{road}$ . The tire is modeled as the spring coefficient,  $K_t$ , and the damping coefficient,  $B_t$ . The suspension of the car locating between the sprung mass and the unsprung mass is modeled as the spring coefficient,  $K_s$ , and the damping coefficient,  $B_s$ . We note that the springs and dampers are characterized as linear system. The vertical velocities of unsprung mass and sprung mass are  $v_u$  and  $v_s$  respectively. Lastly,  $D_u$  and  $D_s$  are the displacements when the springs of unsprung mass and sprung mass are compressed respectively.

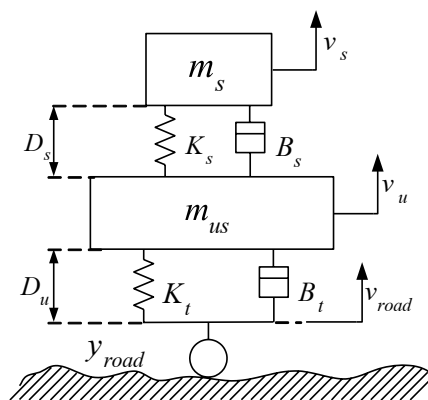


Fig. 1. quarter car model

### 3. Simulink Model

In order to build the quarter car model in Simulink, we use the process as detailed in [6]. According to the Newton's law, all forces are combined, and the result is divided by mass to obtain acceleration as follows:

$$\frac{1}{m_{us}} \sum F = \frac{d^2 D_u}{dt^2} \quad (1)$$

$$\frac{1}{m_s} \sum F = \frac{d^2 D_s}{dt^2} \quad (2)$$

Then the acceleration is integrated to obtain vertical velocity, and the vertical velocity is integrated to obtain the displacement when spring is compressed as follows:

$$\iint \frac{d^2 D_u}{dt^2} = \int \frac{dD_u}{dt} = D_u \quad (3)$$

$$\iint \frac{d^2 D_s}{dt^2} = \int \frac{dD_s}{dt} = D_s \quad (4)$$

We then consider all forces applied to the unsprung mass as detailed in Table 1, and Table 2 is the consideration of all forces applied to the sprung mass. We then get the following equations:

$$0 = -m_{us} \ddot{D}_u + B_t (\dot{y}_{road} - \dot{D}_u) + K_t (y_{road} - D_u) - B_s (\dot{D}_u - \dot{D}_s) - K_s (D_u - D_s) \quad (5)$$

$$0 = -m_s \ddot{D}_s + B_s (\dot{D}_u - \dot{D}_s) + K_s (D_u - D_s) \quad (6)$$

Table 1. Forces considering at unsprung mass

Forces at unsprung mass	equation	direction
From damper of $m_{us}$	$B_t (\dot{y}_{road} - \dot{D}_u)$	+
From spring of $m_{us}$	$K_t (y_{road} - D_u)$	+
From damper of $m_s$	$B_s (\dot{D}_s - \dot{D}_u)$	-
From spring of $m_s$	$K_s (D_s - D_u)$	-

Table 2. Forces considering at sprung mass

Forces at sprung mass	equation	direction
From damper of $m_s$	$B_s (\dot{Z}_s - \dot{Z}_u)$	+
From spring of $m_s$	$K_s (Z_s - Z_u)$	+

Finally, we can construct the Simulink model from the above equations as shown in figure 2.

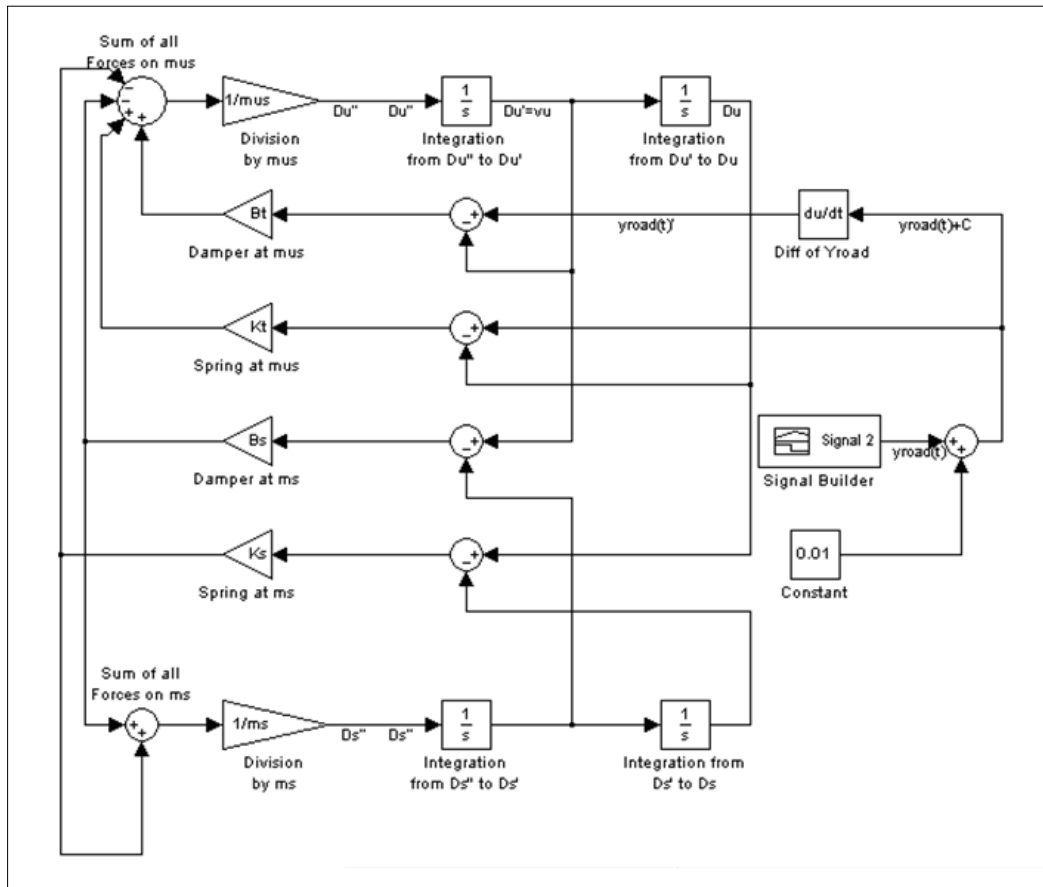


Fig. 2. quarter car model in Simulink

### 3.1. Estimation of a road height

To estimate the road height from the quarter car model, two acceleration sensors are needed to install at sprung mass and unsprung mass of a car as shown in figure 3. Then equations (5) and (6) are rearranged, and we get the following equations:

$$m_{us}\ddot{D}_u + B_s\dot{D}_u - B_s\dot{D}_s + K_s D_u - K_s D_s + B_t\dot{D}_u + K_t D_u = B_t\dot{y}_{road} + K_t y_{road} \quad (7)$$

$$D_s = -\frac{m_s}{K_s}\ddot{D}_s + \frac{B_s}{K_s}\dot{D}_u - \frac{B_s}{K_s}\dot{D}_s + D_u \quad (8)$$

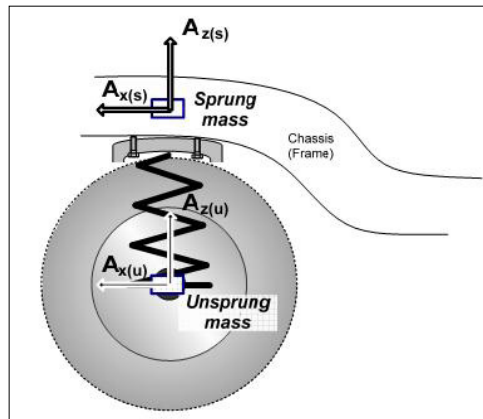


Fig. 3. locations of installing the two acceleration sensors

By replacing equation (8) in equation (7), we then get equation (9) of the road height as follows:

$$\left[ m_{us} \ddot{D}_u + B_t \dot{D}_u + K_t D_u + m_s \ddot{D}_s - B_t \dot{y}_{road} \right] / K_t = y_{road} \quad (9)$$

Therefore, we can construct the model to estimate the road height as shown in figure 4, and the combination of quarter car model and road height estimation model is shown in figure 5.

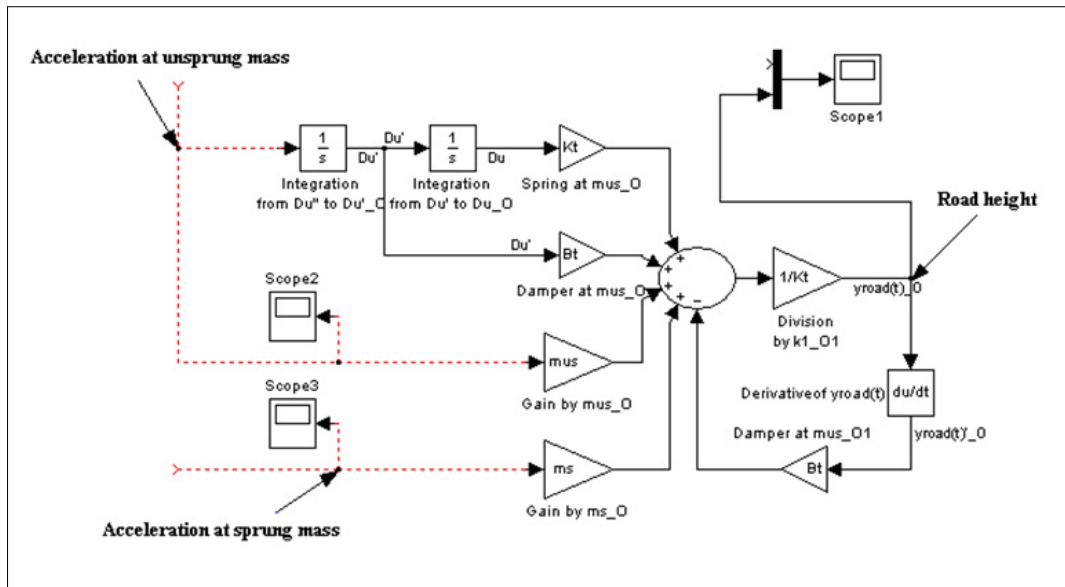


Fig. 4. road height estimation model in Simulink

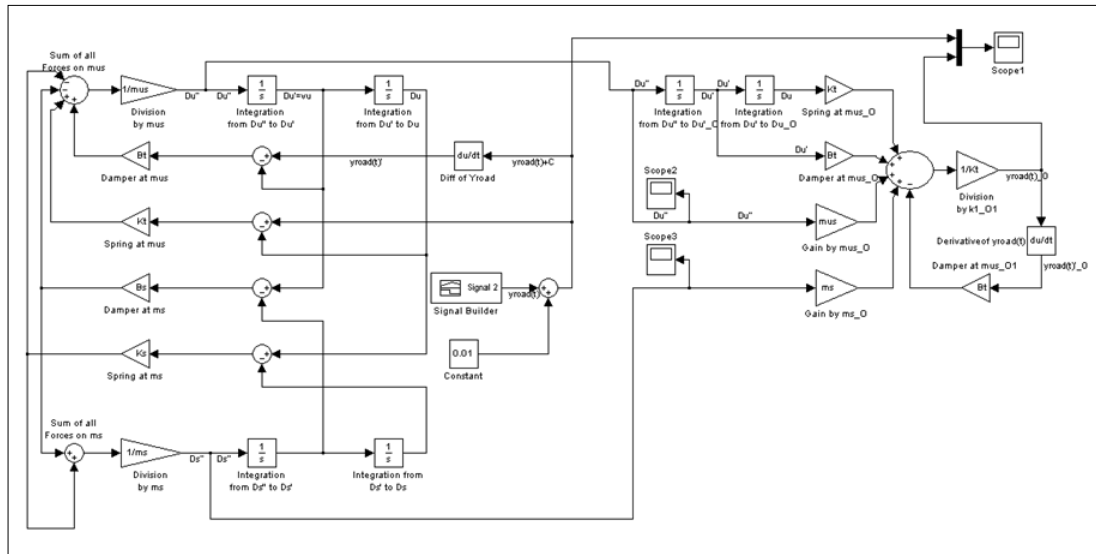


Fig. 5. The combination of quarter car model and road height estimation model

#### 4. Simulation of the road height estimation model

##### 4.1. Parameters of quarter car model and speed hump profile

To simulate the road height estimation model, we use parameters as in [7]. The detail of the parameters is as follows:

$m_s = 454 \text{ kg}$ ,	$m_{ms} = 68 \text{ kg}$ ,	$B_t = 730 \text{ kg/s}$ ,	$B_s = 1627.1 \text{ kg/s}$ ,
$K_t = 256740 \text{ N/m}$	and $K_s = 5565.8 \text{ N/m}$ .		

For the speed hump profile, we use the following equation:

$$z(x) = \begin{cases} -\frac{1}{2}H \left( \cos\left(2\pi \frac{v \cdot t}{L}\right) - 1 \right) & \text{If } 0 < x < L \\ 0 & \text{else} \end{cases} \quad (10)$$

where  $H$  is the height of the hump in cross-section view [m]

$L$  is the width of the hump in cross-section view [m]

$v$  is the velocity of the car passing speed hump [m/s]

$t$  is the elapsed time that the car passed the speed hump [s]

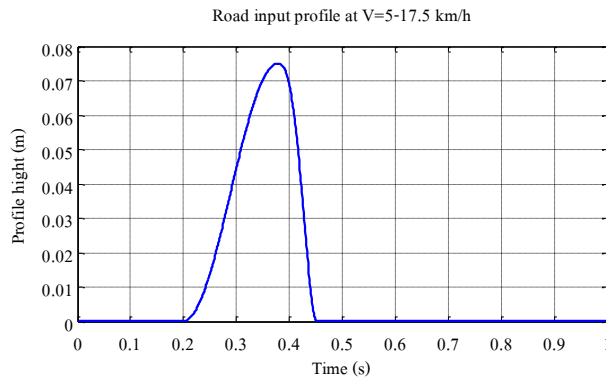


Fig. 6. a speed hump profile created at different velocities

Normally, people drive a car passing a speed hump at different velocities. When encountering a speed hump, people usually slow down the car by putting on the brake and before reaching the top of the speed hump people then put the throttle pedal to increase the car speed. This behaviour must be considered when creating the speed hump profile. Figure 6 shows the result plotted using equation (10) of a car passing a speed hump at the different velocities between 5 km/h – 17.5 km/h, and the speed hump has the height of 0.075 m and width of 0.5 m.

#### 4.2. Simulation results

We use the speed hump profile shown in figure 6 as the input to the quarter car model. Figures 7(a) and 7(b) show the accelerations at unsprung and sprung mass respectively. As seen in the figures, acceleration at unsprung mass fluctuates higher than the acceleration at sprung mass. This is due to the fluctuations that are absorbed by the suspension of the car.

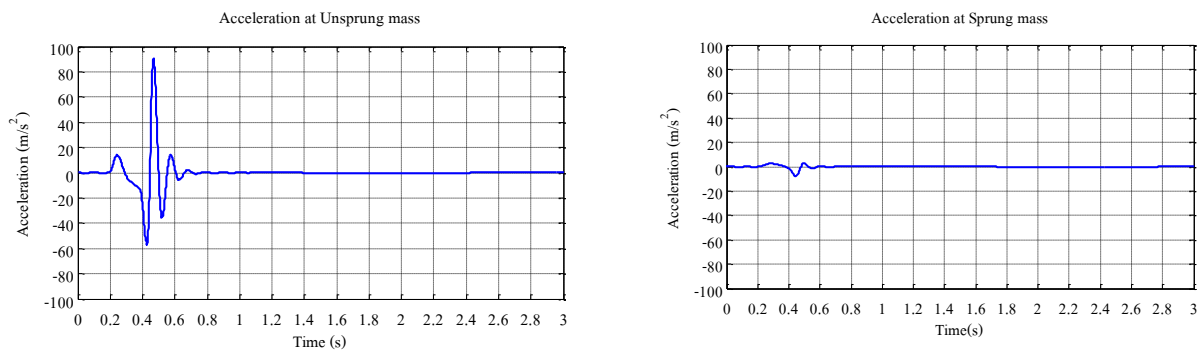


Fig. 7. (a) acceleration at unsprung mass and (b) acceleration at sprung mass

Then the accelerations of the unsprung mass and sprung mass are fed to the road height estimation model. The result is shown in figure 8. As seen in the figure, the estimated profile is the same as the input profile displayed in figure 6.

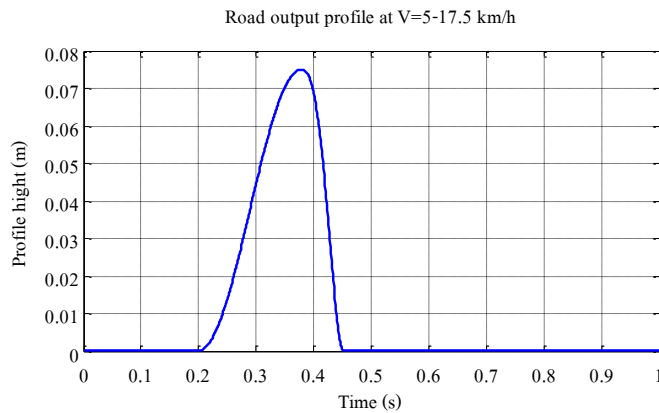


Fig. 8. Estimated road height profile

To estimate the speed hump profile in space unit (height versus distance), the following procedure is needed:

- The velocities while the car passing the speed hump (these velocity information can be obtained from the Electronic Control Unit (ECU) of the car)
- The time period while the car passing the speed hump is divided into time segments
- To change time unit to distance unit in each time segment, we use the equation of  $s = ut$  if the velocity during a time segment is constant and use the equation of  $s = ut + (1/2)at^2$  if there is acceleration during a time segment
- The height in a time segment can then be plotted versus the distance calculated from the previous issue

Figure 9 shows the real speed hump profile (height versus distance) estimated by the above procedure. As seen in the figure, the real speed hump profile is accurately estimated.

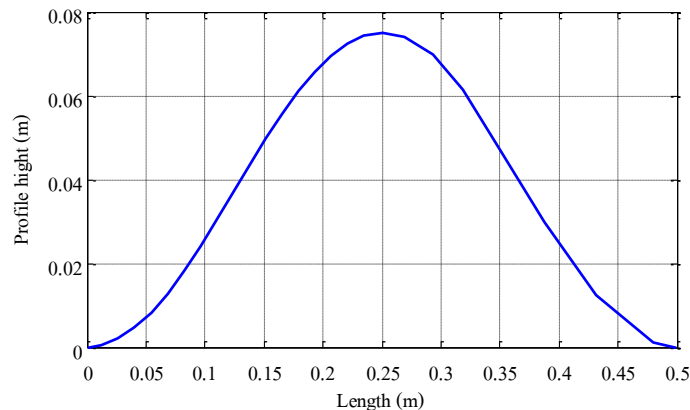


Fig. 9. Estimated speed hump profile



## 5. Conclusion

A technique to estimate a speed hump profile is proposed in this paper. Two acceleration sensors are needed to install at unsprung mass and sprung mass in the proposed technique. We simulated quarter car model and the speed hump estimation model using Simulink. The simulation results show that the speed hump profile can be estimated correctly.

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